A NEW CASE OF CONVECTION IN THE PRESENCE OF COMBINED VERTICAL SALINITY AND TEMPERATURE GRADIENTS

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In water with a combined vertical (z positive upward) temperature gradient (dT/dz) and salinity gradient (dS/dz) we can distinguish four cases:

Case	dT/dz	dS/dz	$d\rho/dz$
1	+	−	−
2 a	+	+	−
2 b	+	+	+
3 a	−	−	−
3 b	−	−	+
4	−	+	+

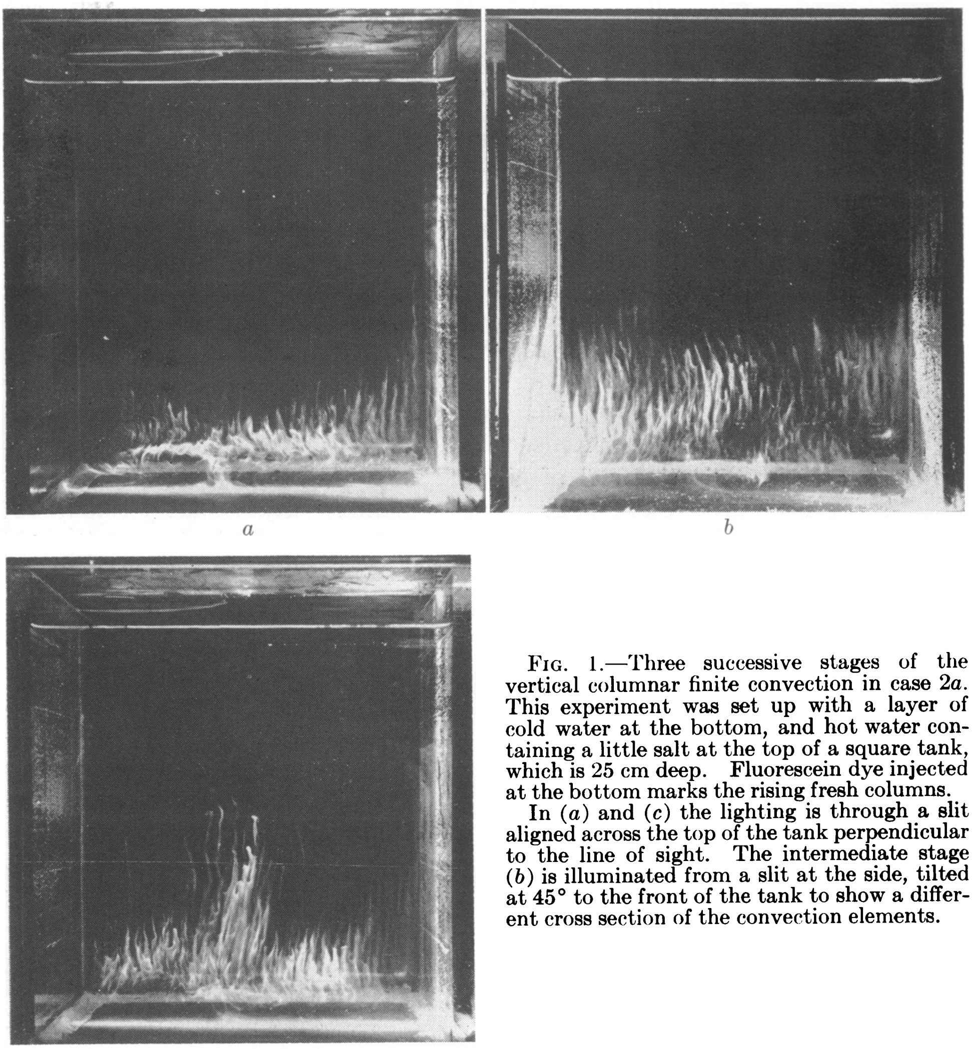
Case 1 is gravitationally stable; the density ρ decreases upward, and no convection can occur. Cases 4, 2*b*, and 3*b* are gravitationally unstable, and correspond to the ordinary Rayleigh convection problem. It is possible to have convection in cases 2*a* and 3*a* even though they are gravitationally stable. A discussion of the corresponding stability problem has been published by Stern.[1] The marginal stability in case 2*a* occurs as exponentially growing cells, and in case 3*a* as overstable oscillatory internal waves of growing amplitude. Because of the experimental difficulties involved in maintaining constant salinity boundary conditions, experiments aimed at verifying the theoretical stability criteria for infinitesimal perturbations have not been made. Experiments exhibiting various forms of finite amplitude convection are easier to perform. The form of finite amplitude convection in case 2*a* is now well known:

The salt convection is highly organized into tall thin columns of fluid alternately ascending and descending. By conducting heat laterally, in the manner of a heat exchanger, the fluid is able to overcome the stabilizing effect of the vertical temperature field, and to draw upon the potential energy in the unstable salt field to drive the convection against viscosity. Once we know the form of the actual convection we can compute the most efficient column diameter. Dr. Willem Malkus of the University of California has formulated an *ad hoc* mathematical theory of this columnar regime which predicts many of the features observed, such as the spacing of the columns and their velocities of ascent and descent.

Figure 1 shows the finite amplitude columnar regime in case 2*a* as photographed from the side, using fluorescein dye to delineate the lower fresh water.

In thinking about what might possibly occur in case 3*a*, we anticipated that a rather different finite amplitude regime would occur (Stommel[2]): in fact, it seemed possible that horizontal layering might occur. Figures 2 and 3 show that this indeed is the case, and we will henceforth confine our attention to this particular form of convection.

The layering shown in Figures 2 and 3 was produced from an initially smooth, stable salinity distribution at constant temperature, by external heating from below. The layers are formed from the bottom up by a turbulent convective mechanism. Elements near the bottom are heated and become buoyant with respect to their immediate surroundings. They rise and mix with the fluid round them until they reach a level where their temperature excess no longer compensates for the density excess due to their high salinity. In this way an initially linear distribution of salinity is well stirred at the bottom, and a layer nearly uniform in temperature and salinity produced. The cessation of the growth of the first layer and the formation of further layers on top of it is a consequence of the difference in the diffusivities



a *b* *c*

FIG. 1.—Three successive stages of the vertical columnar finite convection in case 2*a*. This experiment was set up with a layer of cold water at the bottom, and hot water containing a little salt at the top of a square tank, which is 25 cm deep. Fluorescein dye injected at the bottom marks the rising fresh columns.

In (*a*) and (*c*) the lighting is through a slit aligned across the top of the tank perpendicular to the line of sight. The intermediate stage (*b*) is illuminated from a slit at the side, tilted at 45° to the front of the tank to show a different cross section of the convection elements.

of salt and heat. Heat diffuses through the top of a layer more rapidly than salt, and starts a new convective process above, while the salt stays behind and preserves the stability of the interface.

Each layer is in vigorous turbulent convective motion, with a sharp interface separating it from its neighbors. From time-lapse pictures one can observe that the convective elements are small in scale, about as wide as the depth of the layer,

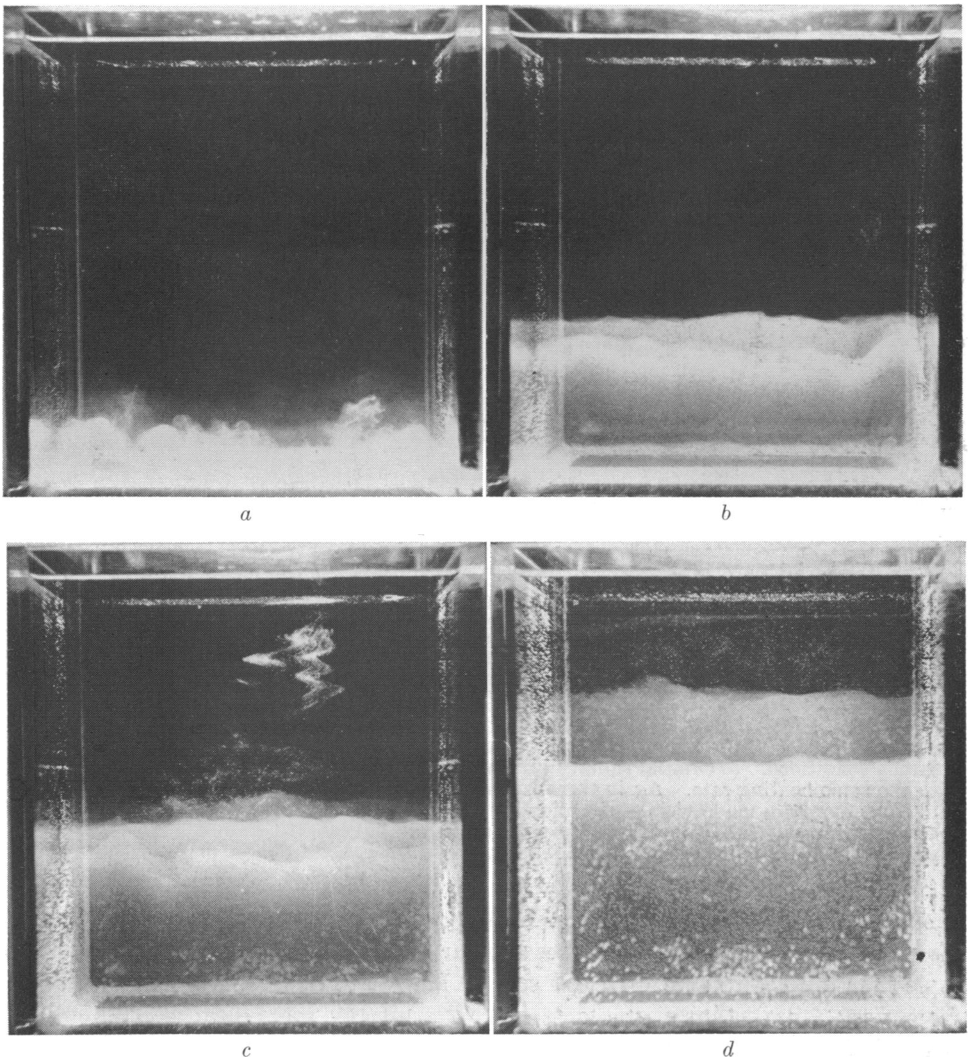
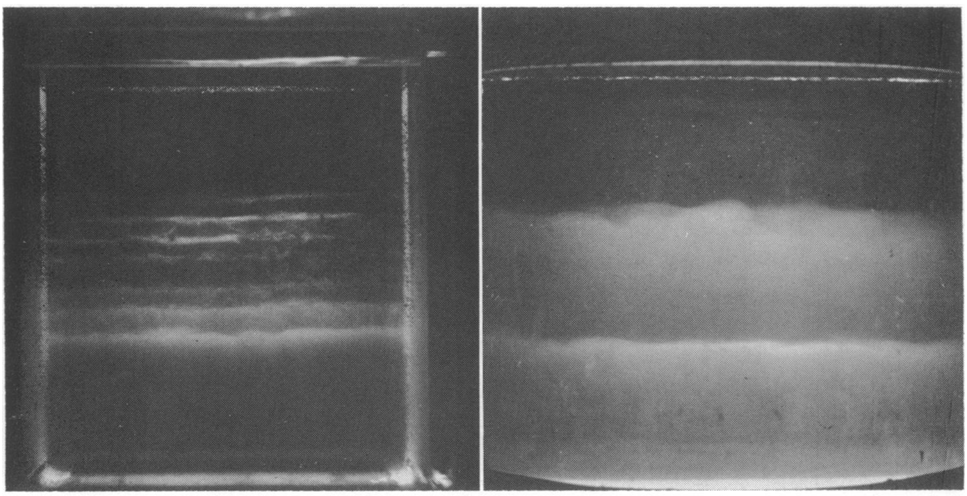


FIG. 2.—Four successive stages of a single experiment, illustrating the stratiform finite amplitude convection in case 3a. Initially there was a smooth gradient of salinity, with about 1% density difference in the 25-cm depth of the tank, and the temperature was uniform. Heating from below at a rate of approximately 2 cals/sq cm/min produced the layered structures shown. All pictures were taken with lighting from a slit across the top of the tank. (a) After 10 min, fluorescein dye injected at the bottom marks a very turbulent layer, with a second forming above it. (b) 25 min: further layers have formed on top and have been successively mixed into the bottom layer. (c) 60 min: three layers are now marked by dye. A streak of aluminum powder injected above these reveals two more turbulent layers, and several more in which the convection is laminar. (d) 95 min: some of the upper layers have merged, leaving three deeper layers.

and random in position. There are no observable organized horizontal motions comparable with the size of the tank. The motions in adjoining layers tend to be in opposite directions near the interfaces, suggesting that elements in the lower layer are moving down along slightly sloping interfaces as they lose their heat, while those in the upper move upward as their temperature rises due to transfer from below. The motions are thermally, not frictionally, driven across the interfaces.

Individual interfaces move upward, but very slowly. The major changes observed with time are the formation of new layers on top of the existing ones, while the two lowest layers become closer in density and eventually merge; the bottom layer thus becomes much thicker than those above it. This behavior can be explained by taking into account the diffusion of salt, as well as heat. Though



a b

FIG. 3.—Further illustrations of the stratiform finite amplitude convection in case 3*a* under different conditions. (*a*) Initial salinity gradient about twice that of the experiment in Fig. 2, with the same heating rate. Again three layers are outlined with fluorescein mixed from below, while four more are made visible with aluminum powder mixed into the water at the beginning of the experiment. (*b*) Deeper layers formed from an initial salinity gradient about the same as that in Fig. 2, with twice the heating rate at the bottom of the tank.

the formation of the layers depends on the greater rate of diffusion of heat, salt is also transferred across the interfaces at a far greater rate than can be explained by molecular diffusion alone. Layers can be formed by this convective mechanism from very large salinity gradients, with a suitably large heating rate. Their depth increases with increasing heating rate and decreases with increasing initial salinity gradient.

The phenomenon is complicated by heat flux horizontally through the walls, and this can independently produce a similar banded structure as shown by Mason and Mendenhall.[3] The relative effect of lateral flux of heat can be reduced by insulation of the side walls and by imposing a strong vertical heat flux. The detailed structure of layers produced in these two ways is different enough to allow one to distinguish between them: lateral heating gives a single large convection cell within each layer, while a heat flux from below produces a smaller scale turbulent motion.

The interfacial phenomena described here are also somewhat similar to the "interfacial turbulence" observed by chemical engineers during the process of solvent extraction (Sherwood and Wei[4]), but the mechanisms are quite different. All their observations relate to the boundary between *immiscible* solvents, and the motion near such interfaces has been explained in terms of the variations of surfa tension with concentration or temperature.

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